

ANALYSIS OF BOND-STIFFNESS DETERIORATION DUE TO DISTRIBUTED DISBONDS

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INTRODUCTION

This paper is concerned with the characterization of residual adhesive strength. The deterioration of adhesive strength depends on damage mechanisms in thin layers at the interfaces of adherends and adhesives. In this work adhesive-bond damage mechanisms, particularly distributions of disbonds, are modeled and related to nonlinear changes of the gross stiffness of the adhesive layer. In turn, the changes in adhesive-layer stiffness affect coefficients of reflection and transmission for reflection and transmission of ultrasonic signals by the adhesive-bond layer. Hence the reflection and transmission measurements can be used to characterize adhesive bond damage.

To investigate the effects of developing disbonding in the adhesive layer, the classical analysis of Goland and Reissner [1] for a lap-joint has been extended to the case that a uniform distribution of disbonds develops in the adhesive layer. Cases of a single disbond, and 10, 20, 30 and an arbitrary number of disbonds uniformly distributed in the adhesive bond have been considered. An interesting result is a plot of the extension of the region of overlap versus the total disbond area for the case of a constant joint-extension force. Another plot displays the shifts in the frequency minima of the reflection coefficient versus the total disbond area. For the latter case theoretical results are shown to agree with experimental results presented in Ref. [2].

THE DISTRIBUTION OF SHEAR AND NORMAL TRACTIONS IN THE ADHESIVE LAYER

For the case that the adhesive layer is modeled by a distribution of tension and shear springs, an analysis of a lap joint was given by Goland and Reissner [1]. Consider two homogeneous linearly elastic thin plates of equal thickness h , joined by an adhesive layer. The length of the bonded part is $2c$. Let P and Q denote the left-side and right-side points of the region of overlap. It is supposed that $h \ll c$. The configuration is shown in Fig. 1. If the lap joint is loaded in tension, the straight line connecting P and Q will rotate from its original unloaded direction due to bending effects. The straight line PQ is defined as the x axis, the origin of the x - y coordinate system is located at the middle point of the line PQ .

When the adhesive layer is represented by layers of tension and shear springs we have

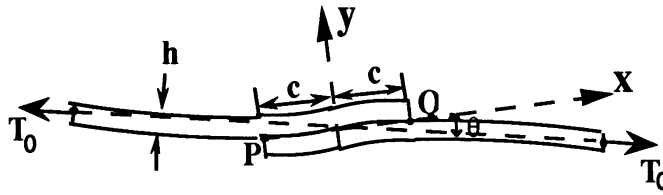


Fig. 1. Configuration of the lap-joint subjected to tensile forces.

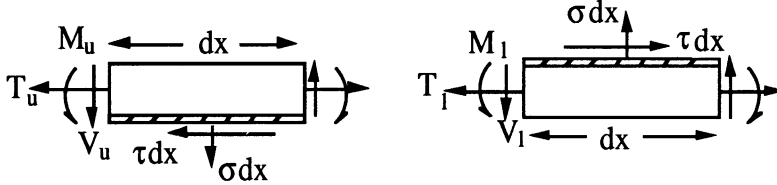


Fig. 2. Sign convention for elements of upper and lower plates.

$$\tau = K_{\tau}(u_u - u_l), \quad \sigma = K_{\sigma}(w_u - w_l), \quad (1a,b)$$

where σ and τ denote the normal and shear tractions on the interfaces, u and w the displacements on the interfaces in the x and the y -directions, K_{τ} and K_{σ} the shear and tension spring constants, respectively, and the subscripts u and l designate quantities pertaining to the upper or lower plate, respectively. Let T , V and M denote the extension force in the x direction, the shear force in the y direction and the bending moment, respectively, each per unit width. Figure 2 shows elements of the upper and lower plates and the adhesive layer, with moments and forces indicated in the positive sense.

Let the magnitude of the external force acting at the left end of the upper plate and at the right end of the lower plate be T_0 and let the angle between the x axis and the line of action of the external forces be θ . Since θ is very small the boundary conditions at $x = \pm c$ can be written approximately as

$$T_u|_{x=-c} = T_l|_{x=c} = T_0, \quad T_l|_{x=-c} = T_u|_{x=c} = 0, \quad (2a,b)$$

$$V_u|_{x=-c} = V_l|_{x=c} = V_0, \quad V_l|_{x=-c} = V_u|_{x=c} = 0, \quad (3a,b)$$

$$-M_u|_{x=-c} = M_l|_{x=c} = M_0, \quad M_l|_{x=-c} = M_u|_{x=c} = 0. \quad (4a,b)$$

Here $V_0 = -T_0 \sin \theta$ and $M_0 = 0.5T_0 h + V_0 c$. Note that in the limit case, if the plates are infinite long in the $\pm x$ directions, we have $V_0 = 0$. For details see Ref. [1].

The differential equations for τ and σ are

$$\frac{d^2\tau}{dx^2} - \alpha^2\tau + \gamma = 0, \quad (5)$$

$$\frac{d^4\sigma}{dx^4} + 4\beta^4\sigma = 0, \quad (6)$$

where

$$\alpha^2 = \frac{8K_\tau}{E_1h}, \quad \gamma = \frac{6K_\tau}{E_1h^2}V_0, \quad \beta^4 = \frac{6K_\sigma}{E_1h^3}. \quad (7a,b,c)$$

Here $E_I = E/(1-\nu^2)$, where E and ν are Young's modulus and Poisson's ratio for the plate material, respectively. The general solution of the ordinary differential equation (5) is

$$\tau = Ae^{\alpha x} + Be^{-\alpha x} + \gamma/\alpha^2, \quad (8)$$

where constants A and B can be determined from the values of $d\tau/dx$ at $x=\pm c$ which are related to T_0 and V_0 . The general solution of the ordinary differential equation (6) is

$$\sigma(x) = C_1f_1(x) + C_2f_2(x) + C_3f_3(x) + C_4f_4(x), \quad (9)$$

where

$$f_1(x) = \cos(\beta x)e^{\beta x}, \quad f_2(x) = \sin(\beta x)e^{\beta x}, \quad (10a,b)$$

$$f_3(x) = \cos(\beta x)e^{-\beta x}, \quad f_4(x) = \sin(\beta x)e^{-\beta x}. \quad (11a,b)$$

The constants C_1 , C_2 , C_3 , and C_4 in (9) can be determined from the values of $d^2\sigma/dx^2$ and $d^3\sigma/dx^3$ at $x=\pm c$ which are related to T_0 and V_0 .

Let $\Delta = u_l(c) - u_u(-c)$ represent the extension of the region of overlap PQ. Then Δ can be expressed in terms of τ and σ as

$$\frac{\Delta}{2c} = \frac{T_0}{2E_1h} + \frac{h}{8cK_\sigma} \left[\frac{d\sigma(c)}{dx} - \frac{d\sigma(-c)}{dx} \right] - \frac{\tau(c) + \tau(-c)}{4cK_\tau}. \quad (12)$$

ADHESIVE LAYER CONTAINING DISBONDS

This section summarizes relevant results. Details can be found in Ref. [3]. Consider the case of a distribution of n disbonds inside the region of overlap. Let the x -coordinates of the left and right tips of the i -th disbond be denoted by s_i and r_i , respectively, and define

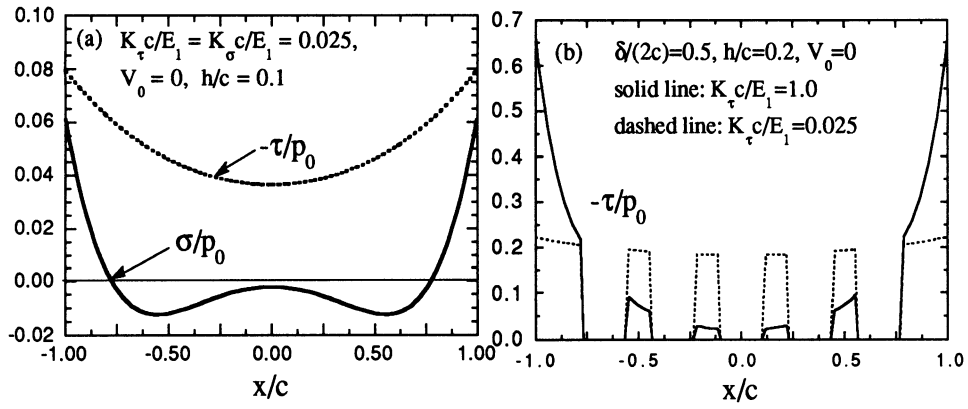


Fig. 3. Distributions of bond-plane tractions: a) without disbands; b) with 5 disbands.

$r_0 = -c$, $s_{n+1} = c$. Note that in the bonded region Eqs. (5) and (6) are still valid. In the debonded regions $\tau=0$ and $\sigma=0$. The shear tractions on the bonded region can be expressed as

$$\tau = A_{i+1}e^{\alpha x} + B_{i+1}e^{-\alpha x} + \gamma/\alpha^2, \quad r_i < x < s_{i+1}, \quad i = 0, 1, 2, \dots, n, \quad (13)$$

where A_{i+1} and B_{i+1} are $2(n+1)$ unknowns which can be determined by the values of $d\tau/dx$ at $x=\pm c$ and the conditions that $u_u(x) - u_l(x)$ and its derivative are continuous at the tips of the disbands. The normal tractions on the bonded regions can be expressed as

$$\sigma(x) = C_1^{i+1}f_1(x) + C_2^{i+1}f_2(x) + C_3^{i+1}f_3(x) + C_4^{i+1}f_4(x), \quad r_i < x < s_{i+1}, \quad i = 0, 1, 2, \dots, n, \quad (14)$$

where C_1^{i+1} , to C_4^{i+1} are $4(n+1)$ unknown constants which can be determined by the values of $d^2\sigma/dx^2$ and $d^3\sigma/dx^3$ at $x=\pm c$ and the conditions that $w_u(x) - w_l(x)$ and its first to third derivatives are continuous at the tips of the disbands. The extension of the region of overlap can be expressed as

$$\begin{aligned} \frac{\Delta}{2c} = & \frac{T_0}{2E_1h} - \frac{\tau(c) + \tau(-c)}{4cK_\tau} + \frac{h}{8cK_\sigma} \left\{ \frac{d\sigma(c)}{dx} - \frac{d\sigma(-c)}{dx} + \right. \\ & \left. \sum_{i=1}^n \left[\frac{d\sigma(s_i)}{dx} - \frac{d\sigma(r_i)}{dx} + \frac{d^2\sigma(s_i)}{dx^2}(r_i - s_i) + \frac{1}{2} \frac{d^3\sigma(s_i)}{dx^3}(r_i - s_i)^2 \right] \right\}. \end{aligned} \quad (15)$$

For $K_\tau c/E_1 = 0.025$, $K_\sigma c/E_1 = 0.025$, $V_0 = 0$ and $h/c = 0.1$, the distributions of shear and normal tractions on the bond-plane without disbands are shown in Fig. 3a, where the tractions are normalized by $p_0 = T_0/h$. It is noted that both shear and normal tractions reach their maximum values at the ends of the region of overlap. For $V_0 = 0$, $h/c = 0.2$, the shear traction distributions on the bond-plane with 5 disbands for $K_\tau c/E_1 = 0.025$ and $K_\tau c/E_1 = 1.0$ are shown in Fig. 3b. It is found that the traction concentrations at the ends of the region of overlap are more prominent for a stiffer adhesive layer.

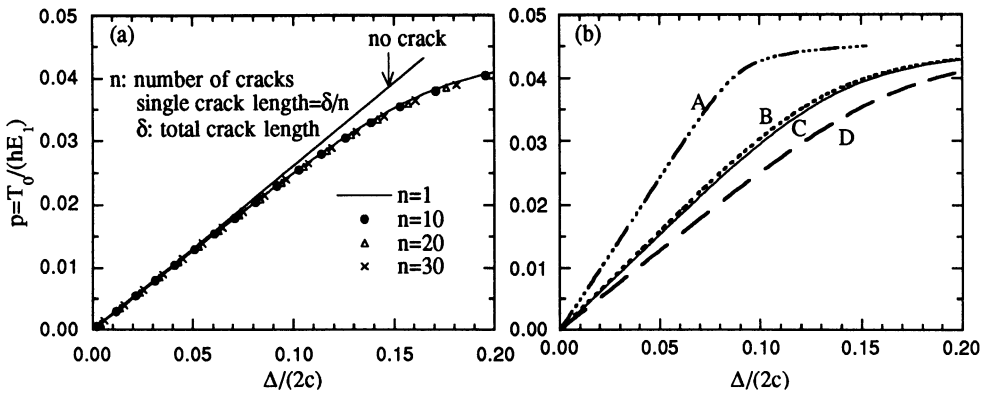


Fig. 4. Plots of dimensionless load versus extension of the region of overlap. (a) for various number of disbonds and $K_\tau c/E_I = K_\sigma c/E_I = 0.025$, $V_0 = 0$, $h/c = 0.2$, (b) for various values of the spring constants, A: $K_\tau c/E_I = K_\sigma c/E_I = 0.1$, $V_0 = 0$; B: $K_\tau c/E_I = 0.025$, $K_\sigma c/E_I = 100$, $V_0 = 0.02$; C: $K_\tau c/E_I = 0.025$, $K_\sigma c/E_I = 100$, $V_0 = 0$; D: $K_\tau c/E_I = K_\sigma c/E_I = 0.1$, $V_0 = 0$.

NONLINEAR RELATION BETWEEN LOAD AND EXTENSION OF THE REGION OF OVERLAP

Since the system of governing equations is linear, the bond-plane tractions, τ and σ are proportional to the load T_0 for a given distribution of disbonds. Thus Eq. (5) indicates that the extension of the region of overlap, $\Delta / (2c)$ is linear with load T_0 for a given distribution of disbonds. On the other hand, the extension of the region of overlap is a complicated function of the distribution of the disbonds. Let δ denote the total length of the disbonds. A simple model of the disbond distribution is that there are n uniformly distributed disbonds of the same length. Then

$$s_i = 0.5(2c - \delta) / n + (2c / n)(i - 1), \quad r_i = s_i + \delta / n, \quad i = 1, 2, \dots, n. \quad (16)$$

For a specific computation, we use the following disbond-area-load relation ,

$$\begin{aligned} \delta(2c) &= 0, & p < 0.01 \\ &= 0.2(p - 0.01) / (0.05 - p), & 0.01 < p < 0.05, \end{aligned} \quad (17)$$

where $p = T_0 / (hE_I)$ is the dimensionless load. For $n=0, 1, 10, 20$ and 30 , the relation between p and $\Delta / (2c)$ is shown in Fig. 4a. It is interesting to note that the curves almost coincide for $n=1$ to 30 . For different spring constants of the adhesive layer the p - $\Delta / (2c)$ curves are shown in Fig. 4b. The small difference between curves B and C in Fig. 4b indicates that the shear force V_0 does not significantly affect the relation between p and

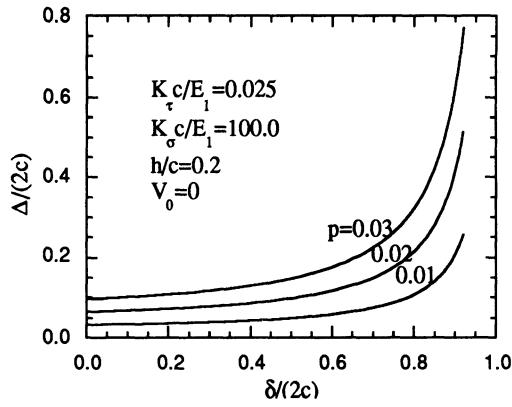


Fig. 5. Plots of extensions of the region of overlap versus percentage disbond area for 3 values of p .

$\Delta/(2c)$. For fixed p , $\Delta/(2c)$ is a function of $\delta/(2c)$. For three values of p , the curves of $\Delta/(2c)$ versus $\delta/(2c)$ are shown in Fig.5.

For an adhesive layer containing disbands we define the equivalent shear spring constant as

$$\bar{K}_\tau = \int_{-c}^c \tau(x) dx / \int_{-c}^c [u_u(x) - u_l(x)] dx, \quad (18)$$

which can be expressed in terms of τ by

$$\bar{K}_\tau = \left\{ \frac{1}{K_\tau} - \frac{1}{T_0 K_\tau} \sum_{i=1}^n \left[\tau(s_i)(r_i - s_i) + \frac{1}{2} \frac{d\tau(s_i)}{dx} (r_i - s_i)^2 - \frac{\gamma}{6} (r_i - s_i)^3 \right] \right\}^{-1}. \quad (19)$$

Since the shear traction τ and γ defined by Eq. (7b) are proportional to the load T_0 , the expression on the right hand side of (19) indicates that \bar{K}_τ is independent of T_0 .

DETERMINATION OF ADHESIVE BOND DETERIORATION BY MEASURING THE SHIFT OF THE FREQUENCY MINIMA OF THE REFLECTION COEFFICIENT

Consider two semi-infinite linear elastic solids with Lamé elastic constants λ, μ and mass density ρ , joined by a thin layer of adhesive material with Lamé elastic constants $\bar{\lambda}_0$, $\bar{\mu}_0$ and mass density $\bar{\rho}_0$. The configuration is shown in Fig. 6. It is well known that as a function of the frequency the reflection coefficient of the TV wave displays minima whose

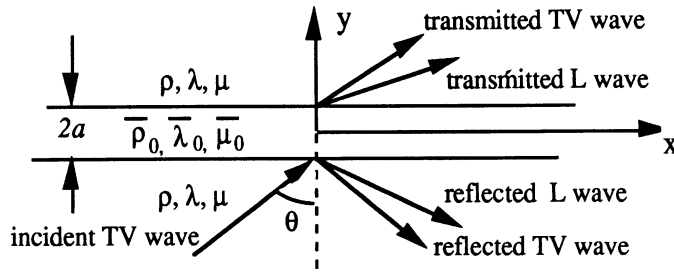


Fig. 6. TV wave obliquely incident on the adhesive layer.

positions depend on the material properties of the two half spaces and the adhesive layer, and on the thickness of the adhesive layer. For the cases without and with disbonds we have

$$K_{\tau} = \bar{\mu}_0 / (2a), \text{ and } \bar{K}_{\tau} = \bar{\mu} / (2a), \quad (20a,b)$$

respectively, where \bar{K}_{τ} as a function of K_{τ} and the disbond area was defined by Eq. (19). Thus $\bar{\mu}$ can be calculated by substituting (19) into (20b). For simplicity we assume that $\bar{\lambda} / \lambda_0 = \bar{\mu} / \mu_0$. The mass in the debonded region is $2a\delta\bar{\rho}_0$, where $2a$ is the thickness of the adhesive layer. If this mass does not take part in the dynamic motion, the average mass density of the damaged adhesive layer may be expressed as $[1-\delta/(2c)]\bar{\rho}_0$. When waves propagate in the debonded adhesive layer, part of the mass in the debonded region still affects, however, the dynamic response. Thus it is assumed that the equivalent mass density of the damaged adhesive layer can be expressed as $\bar{\rho} = [1-\varepsilon\delta/(2c)]\bar{\rho}_0$, where $0 < \varepsilon < 1$. In the case of obliquely incident TV wave, the reflection coefficient of TV wave can be obtained numerically. Thus for a given area of disbonds δ , the reflection coefficients can be calculated with ε as a parameter. We will choose the value of the constant ε such that the numerical results agree best with the experimental ones.

The theoretical results are compared with experimental results obtained by Rokhlin et al. [2] for an adhesive bond that has been subjected to moisture infiltration. The moisture appears to first infiltrate the adhesive layer and then attack the adhesive bond, to generate a distribution of disbonds. In the experimental work of Rokhlin et al. [2] $h=1.594\text{mm}$, $2c=1.27\text{cm}$, $2a=0.147\text{mm}$, $c_L=6.32\text{km/sec}$, $c_T=3.143\text{km/sec}$, $\rho=2.7\text{g/cm}^3$,

$\bar{c}_{L0}=2.1\text{km/sec}$, $\bar{c}_{T0}=1\text{km/sec}$, $\bar{\rho}_0=1.5\text{g/cm}^3$. The corresponding dimensionless

equivalent shear spring constant of the adhesive layer is shown in Fig. 7 as a function of the percentage disbond area. It is noted that a practically linear relation between \bar{K}_{τ}/K_{τ} and $\delta/(2c)$ is obtained. For a TV wave with incident angle $\theta=34.4^\circ$ the calculated and experimental results are shown in Fig. 8. Note that in ref.[2], the shifts of the frequency minima of reflection coefficients are related to the frequency at which the minimum reflection coefficient occurs before moisture infiltration, say f_0 . Let f_1 denote the frequency at the minimum reflection coefficient after moisture infiltration but prior to disbond generation. The experimental result for f_1-f_0 is about 0.335MHz. In Fig. 8 the shifts of the frequency minima are to f_1 . Good agreement between theory and experiment is obtained

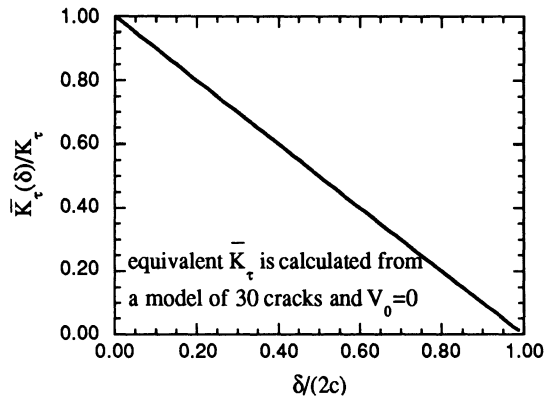


Fig. 7. Plot of equivalent \bar{K}_τ/K_τ versus percentage disbond area.

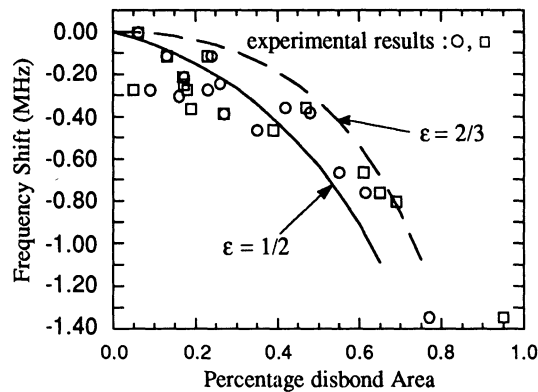


Fig. 8. Shifts in the frequency minima of reflection coefficients versus percentage disbond area.

for $1/2 < \epsilon < 2/3$. The results displayed in Fig. 8 indicate that a measurement of the frequency shift can provide the percentage of disbond area. This percentage is directly related to the residual strength of the adhesive layer.

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